Order-Invariant Real Number Summation

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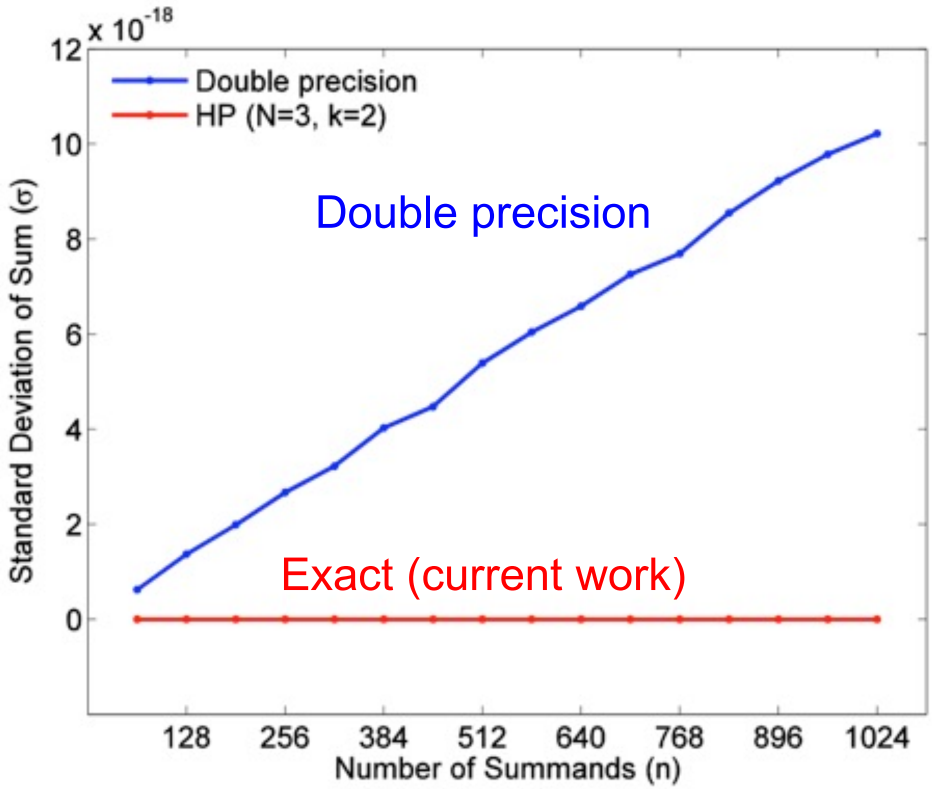
P. E. Small *et al.*, *Proc. IEEE IPDPS*, p. 152 ('16)
https://aiichironakano.github.io/cs596/Small-OrderInvariantSum-IPDPS16.pdf

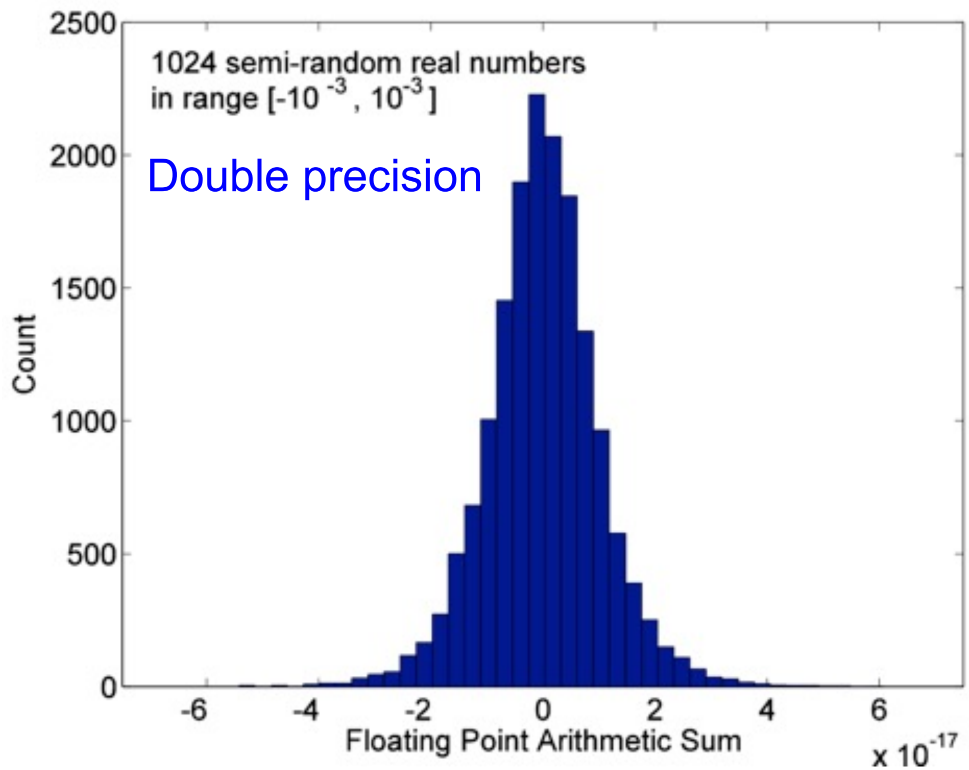
Reproducibility Challenge

- **Rounding (truncation) error makes floating-point addition non-associative**

$$(a + b) + c \neq a + (b + c)$$



Standard deviation of sum with random summation orders



Distribution of sum with random summation orders

- **Finding: Sum becomes a random walk across the space of possible rounding error**

Solution: High-Precision (HP) Method

- **Propose an extension of the order-invariant, higher-precision intermediate-sum method by Hallberg & Adcroft** [*Par. Comput*. **40**, 140 ('14)]

- **The proposed variation represents a real number *r* using a set of *N* 64-bit unsigned integers, $\boldsymbol{a_i}$ ($\boldsymbol{i \in [0, N-1]}$)**

$$r = \sum_{i=0}^{N-1} a_i 2^{64(N-k-i-1)}$$
$$= \overbrace{a_0 2^{64(N-k-1)} + \cdots + a_{N-k-1}}^{N-k} + \overbrace{a_{N-k} 2^{-64} + \cdots + a_{N-1} 2^{-64k}}^{k}$$

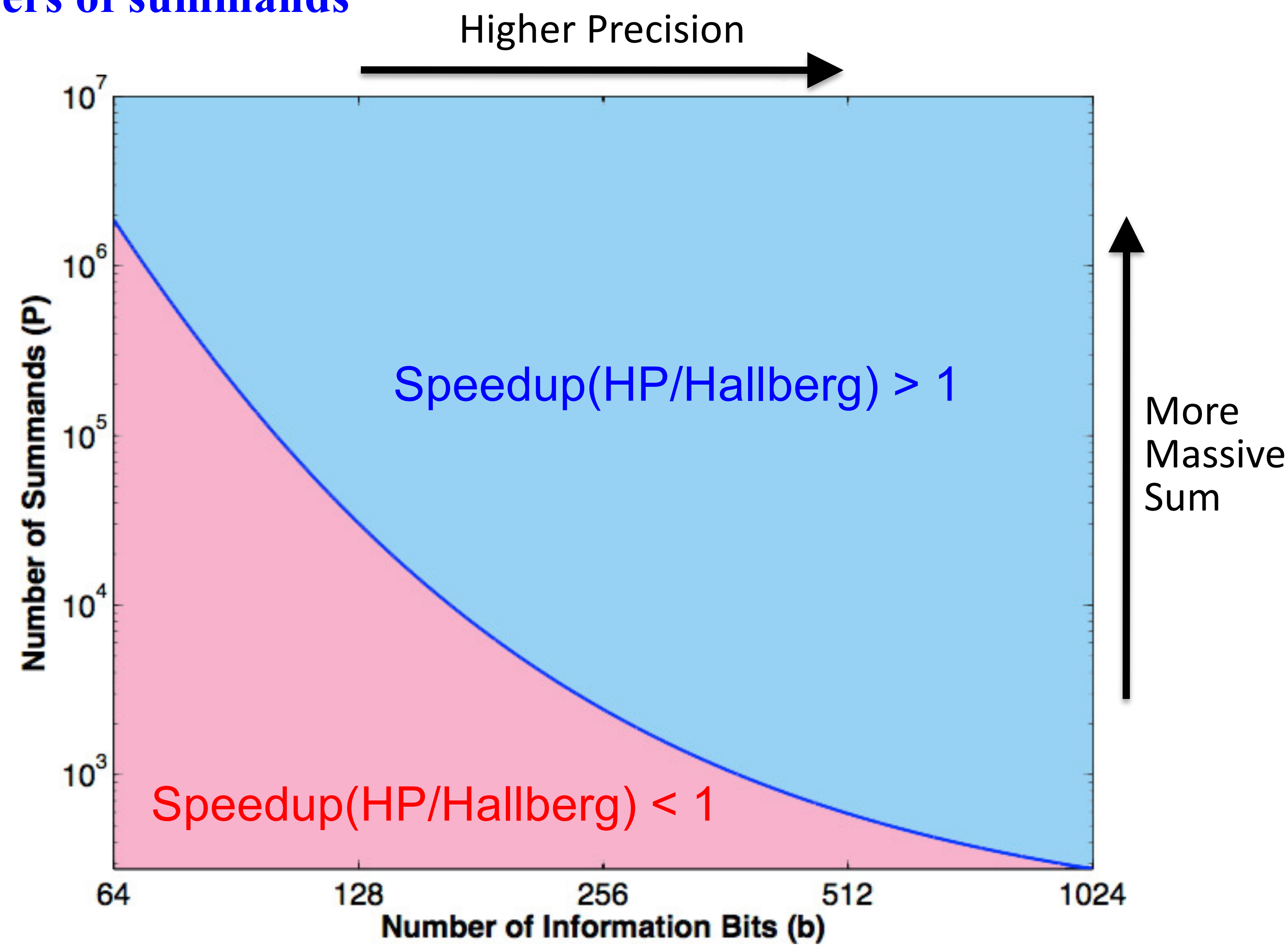
- ***k* is the number of 64-bit unsigned integers assigned to represent the *fractional* portion of *r* ($0 \le k \le N$), whereas *N*-*k* integers represent the *whole-number* component**

- **Negative number is represented by two's complement in integer representation, using only 1 bit**

If you are the first to find the problem, the simplest solution suffices to prove the concept

Performance Projection

- **HP sum is faster than Hallberg sum for higher precision & larger numbers of summands**



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