Multiresolution Analysis Using Wavelets

HAAR BASIS

Consider a one dimensional "image" on 2 pixels: `I[2]` = (6, 2). We decompose this information into a "smooth" and a "detailed" components. The smooth component is an "average" of the two intensities:

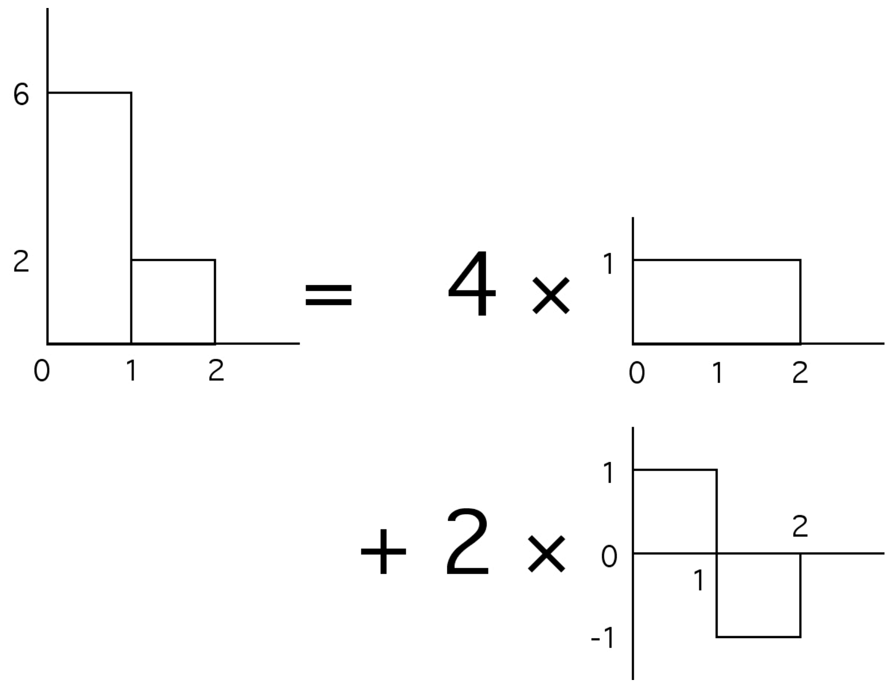
$$s = (6 + 2)/2 = 4$$

The detailed component is a "difference" between the two intensities:

$$d = (6 - 2)/2 = 2$$

We can think of this decomposition as a linear combination of two one-dimensional vectors:

$$\vec{I} = (6,2) = 4 \times (1,1) + 2 \times (1,-1)$$



The function

$$\phi(x) = 1\ (0 \le x < 2);\ 0\ \text{(otherwise)},$$

is called the *Haar scaling function*. The detailed component is represented by another function called the *Haar wavelet*:

$$\psi(x) = 1\ (0 \le x < 1);\ -1\ (1 \le x < 2);\ 0\ \text{(otherwise)}.$$

WAVELET DECOMPOSITION

Consider a one dimensional "image" `I`[16] on 16 pixels:

```
I[16]  1  2  5  9  1  9  2  2  2  3  5  7  4  2  1  1
```

We define "smooth" components whose number is half the number of the original image:

```
s[i] = (I[2*i] + I[2*i+1])/2 (i = 0,...,7)
```

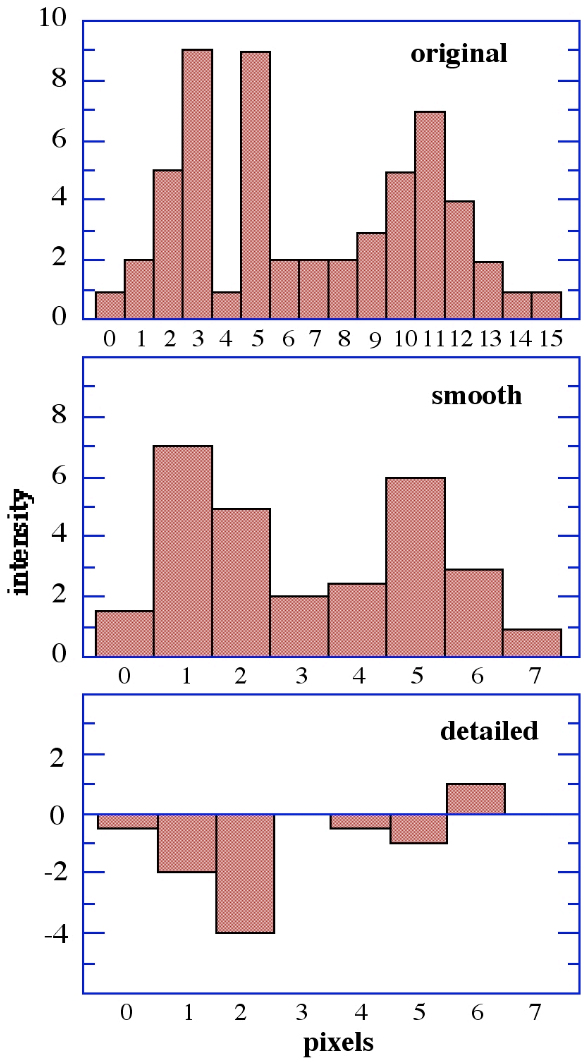
Half of the information contained in the original image can be represented by the "detailed" components, which are defined as,

```
d[i] = (I[2*i] - I[2*i+1])/2 (i = 0,...,7)
```

Combination of the "smoothed" and "detailed" images contains the same amount of information as the original image.

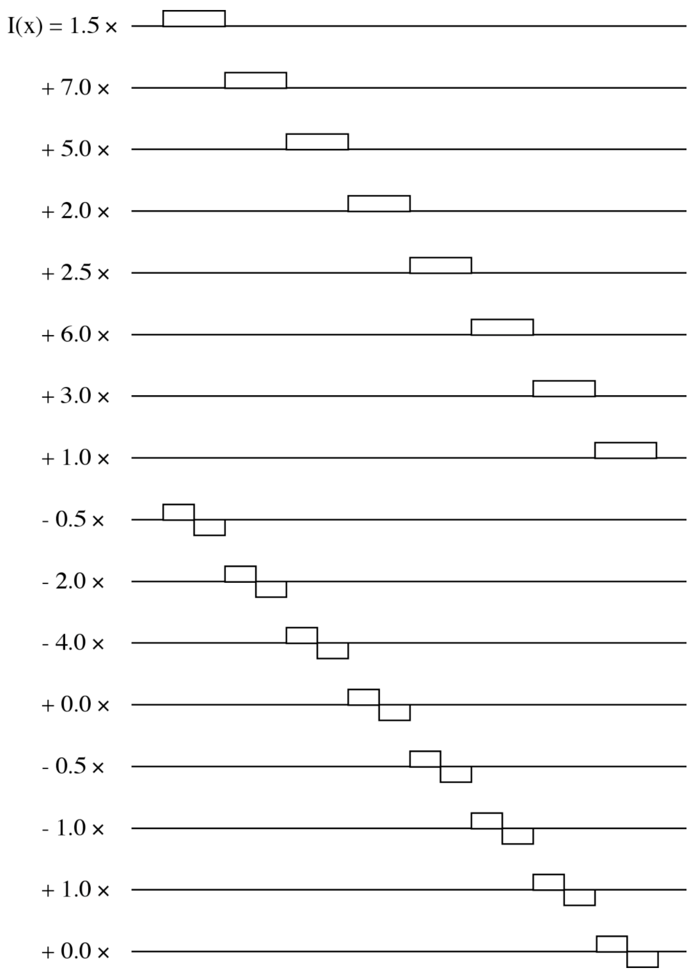
I[16]	1	2	5	9	1	9	2	2	2	3	5	7	4	2	1	1
s[8]	1.5		7.0		5.0		2.0		2.5		6.0		3.0		1.0	
d[8]		-0.5		-2.0		-4.0		0.0		-0.5		-1.0		1.0		0.0

WAVELET DECOMPOSITION OF A ONE-DIMENSIONAL IMAGE



We can think of this decomposition as a linear combination of "shifted" scaling functions and wavelets: The vector space v^0 is now decomposed into the "smooth" subspace v^1 and the "detailed" subspace w^1.

$$v^0 = v^1 + w^1$$



MULTIRESOLUTION ANALYSIS

The above decomposition can be applied recursively. The “smoothed” representation `s[8]` $\in V^1$ still contains 8 values which can be decomposed into “further smoothed” components `ss[4]` $\in V^2$ and “detailed” components `sd[4]` $\in W^2$. This is continued as:

```
I[16] -> s[8], d[8]
          s[8] -> ss[4], sd[4]
                  ss[4] -> sss[2], ssd[2]
                           sss[2] -> ssss[1]. sssd[1]
```

or

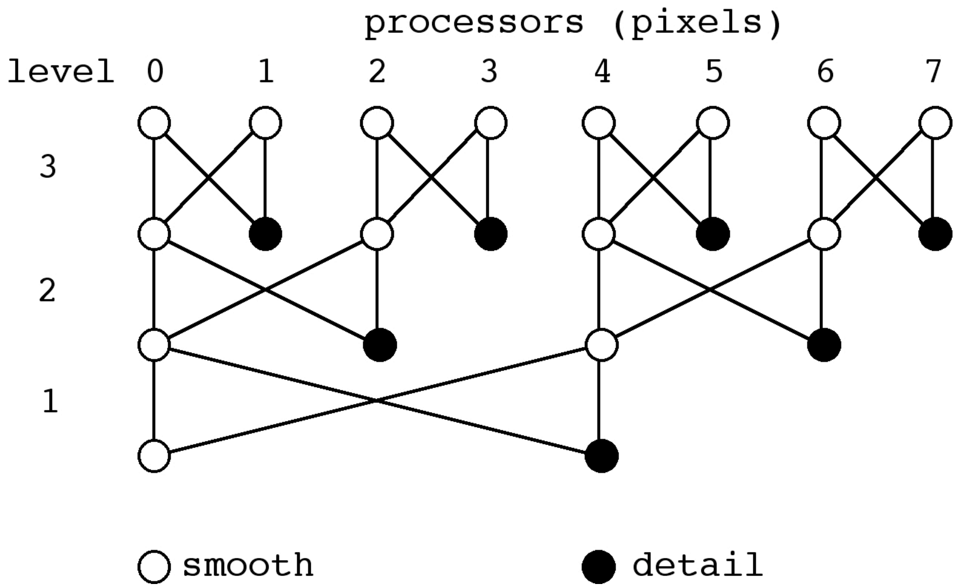
$$
\begin{aligned}
V^0 &= V^1 + W^1 \\
&= V^2 + W^2 + W^1 \\
&= V^3 + W^3 + W^2 + W^1 \\
&= V^4 + W^4 + W^3 + W^2 + W^1
\end{aligned}
$$

The multiresolution representation of the image thus consists of the hierarchy of “detailed” representations and the coarsest “smooth” representation.

```
I[16] -> d[8], sd[4], ssd[2], sssd[1], ssss[1]
```

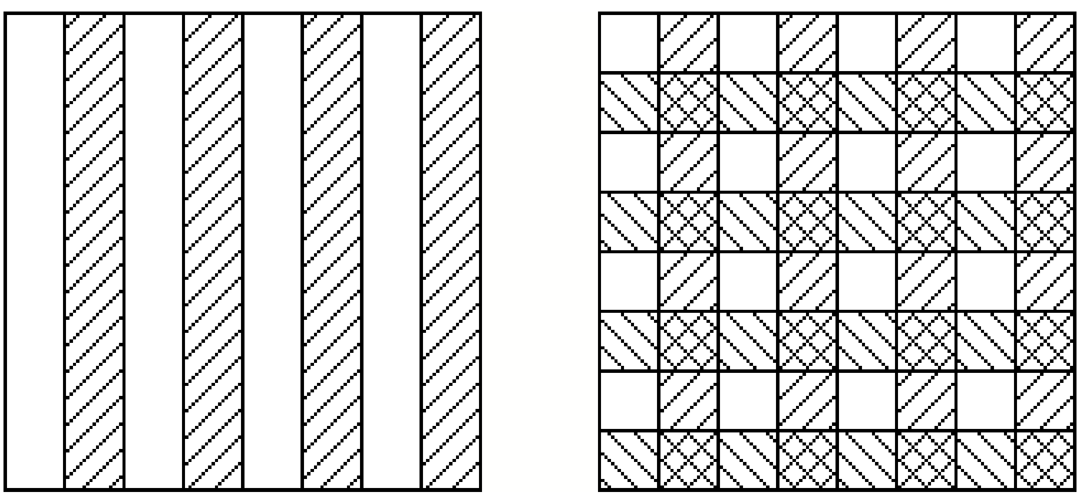
PARALLEL MULTIRESOLUTION ANALYSIS

We use a tournament communication pattern (see the figure below). Communication stride is doubled at each level. Even nodes calculate a smoothed component; odd nodes calculate a detailed component. Only even nodes go to the next level.

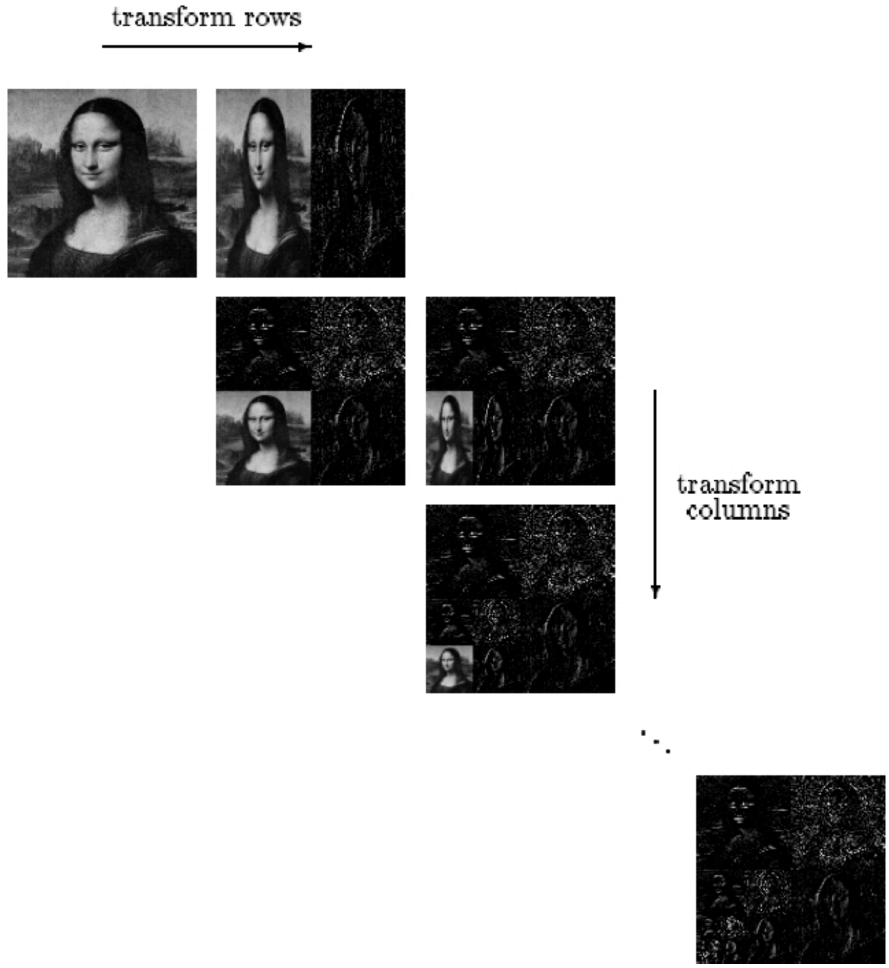


TWO-DIMENSIONAL IMAGE COMPRESSION

Consider a 2-dimensional image on $2^{L\mathrm{max}}$ by $2^{L\mathrm{max}}$ pixels. We compress this image into an image on $2^{L\mathrm{min}}$ by $2^{L\mathrm{min}}$ pixels. The program has an outermost for loop over $l = L\mathrm{max}$ down to $L\mathrm{min}+1$. At each level, we perform one row and one column decomposition.



MULTIRESOLUTION IMAGE COMPRESSION



Reference

1. "Wavelets for computer graphics: a primer," E. J. Stollnitz, T. D. DeRose, and D. H. Salesin, *IEEE Computer Graphics Appl*. **15**(3), 76 (1995).