Discussion: density matrix renormalization group

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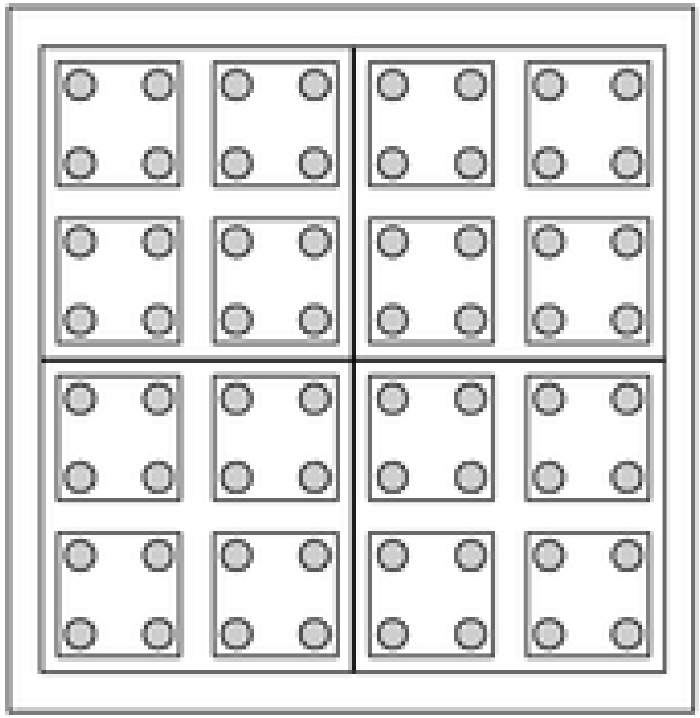
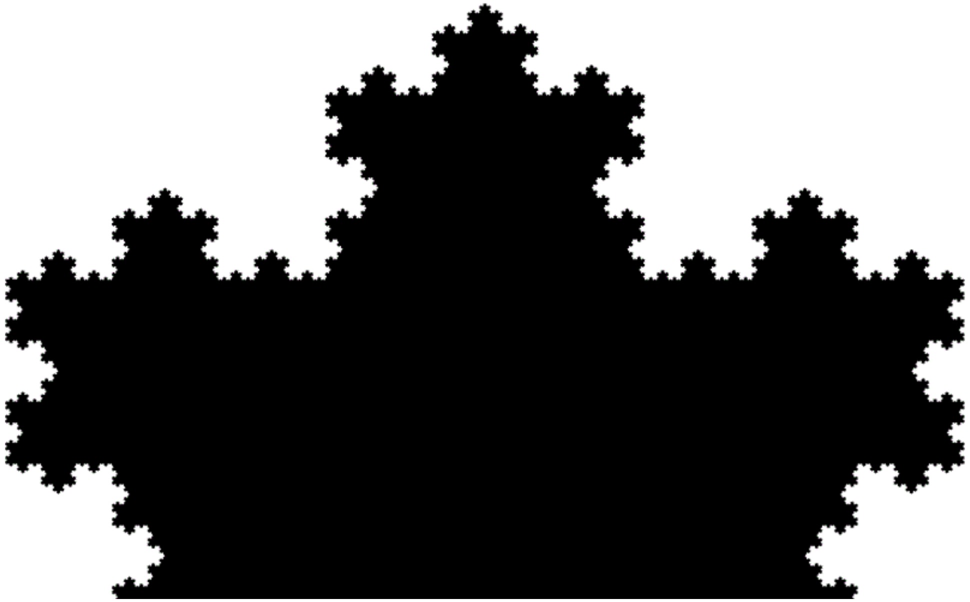
CSCI 699

Renormalization group



System: Self-similarity

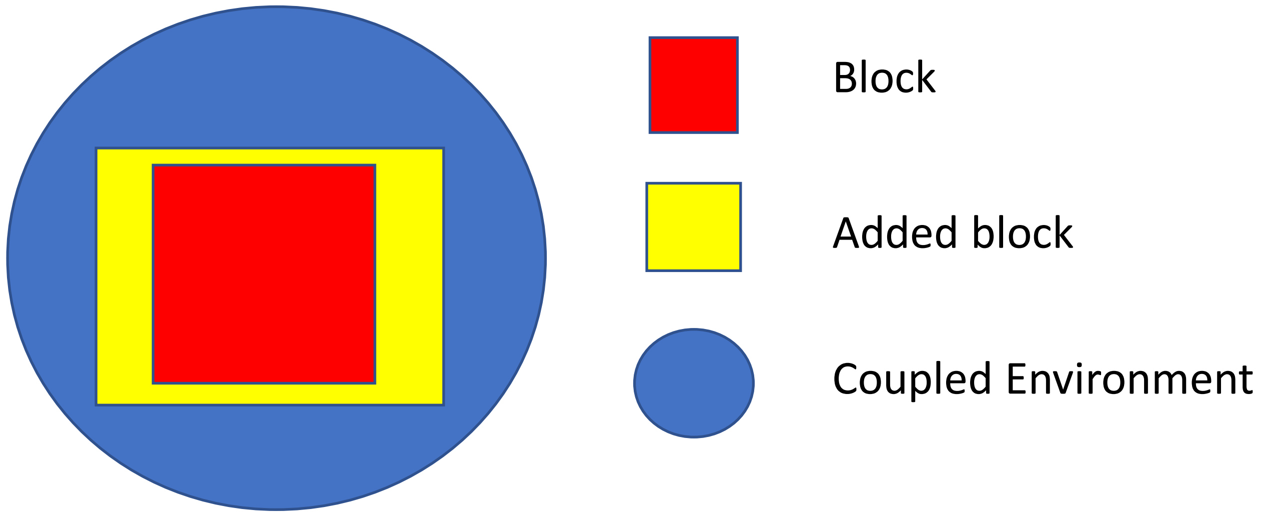


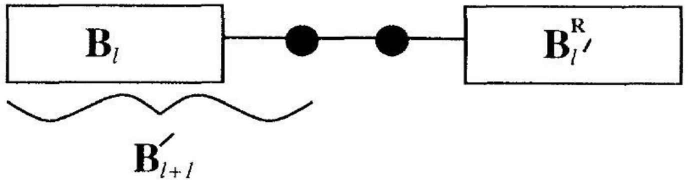
$$-\beta H = J\sum_{i,j} s_i s_j + h\sum_i s_i$$

scale transformation:

$$-\beta H = J_b\sum_{i,j} S^b{}_i S^b{}_j + h_b\sum_i S^B{}_i$$

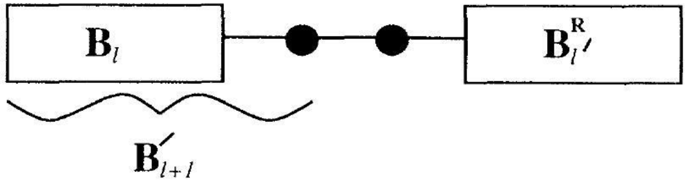
Ideas: renormalization of many-body problem



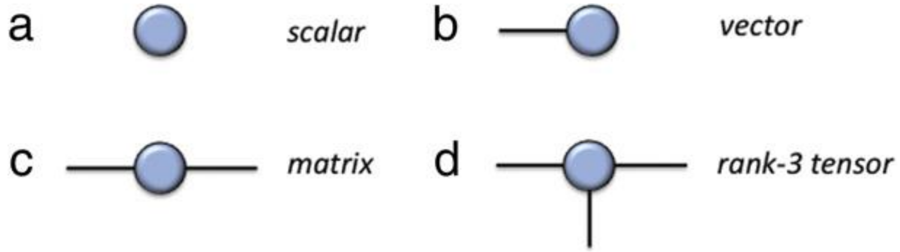


B_l	Block consisting of m states
B'_{l+1}	Block + one site, no outside coupling
$\bullet B^R_{l'}$	Environments
B_{l+1}	New block, using previous truncated density matrix methods

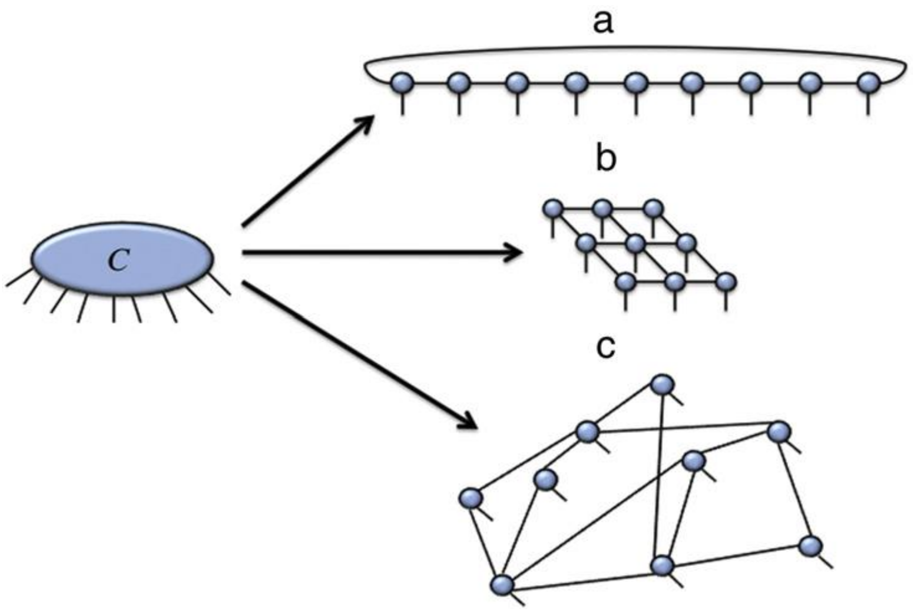
Tensor network



Tensor



Tensor network decomposition



1D tensor network: Matrix product states

$$|\Psi\rangle = \sum_{\alpha_1=1}^{\min(p,D)} \lambda^{[1]}_{\alpha_1} |\tau^{[1]}_{\alpha_1}\rangle \otimes |\tau^{[2\cdots N]}_{\alpha_1}\rangle,$$

$$|\Psi\rangle = \sum_{i_1=1}^{p} \sum_{\alpha_1=1}^{\min(p,D)} \Gamma^{[1]i_1}_{\alpha_1} \lambda^{[1]}_{\alpha_1} |i_1\rangle \otimes |\tau^{[2\cdots N]}_{\alpha_1}\rangle,$$

$$|\Psi\rangle = \sum_{i_1=1}^{p} \sum_{\alpha_1=1}^{\min(p,D)} \Gamma^{[1]i_1}_{\alpha_1} \lambda^{[1]}_{\alpha_1} |i_1\rangle \otimes |\tau^{[2\cdots N]}_{\alpha_1}\rangle,$$

$$|\Psi\rangle = \sum_{i_1,i_2=1}^{p} \sum_{\alpha_1=1}^{\min(p,D)} \sum_{\alpha_2=1}^{\min(p^2,D)} \left(\Gamma^{[1]i_1}_{\alpha_1} \lambda^{[1]}_{\alpha_1} \Gamma^{[2]i_2}_{\alpha_1\alpha_2} \lambda^{[2]}_{\alpha_2}\right) |i_1\rangle \otimes |i_2\rangle \otimes |\tau^{[3\cdots N]}_{\alpha_2}\rangle.$$

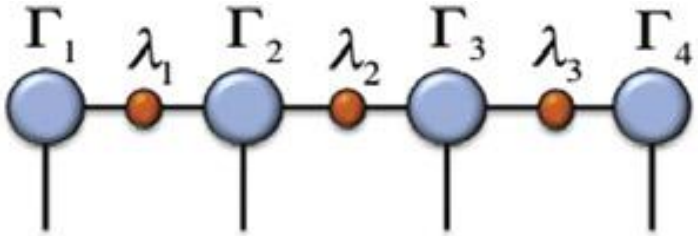
$$|\Psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} \left(\Gamma^{[1]i_1}_{\alpha_1} \lambda^{[1]}_{\alpha_1} \Gamma^{[2]i_2}_{\alpha_1\alpha_2} \lambda^{[2]}_{\alpha_2} \ldots \lambda^{[N-1]}_{\alpha_{N-1}} \Gamma^{[N]i_N}_{\alpha_{N-1}}\right) |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle,$$

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

Matrix product states

Canonical form

$$|\Psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} \left(\Gamma^{[1]i_1}_{\alpha_1} \lambda^{[1]}_{\alpha_1} \Gamma^{[2]i_2}_{\alpha_1\alpha_2} \lambda^{[2]}_{\alpha_2} \dots \lambda^{[N-1]}_{\alpha_{N-1}} \Gamma^{[N]i_N}_{\alpha_{N-1}} \right) |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle,$$



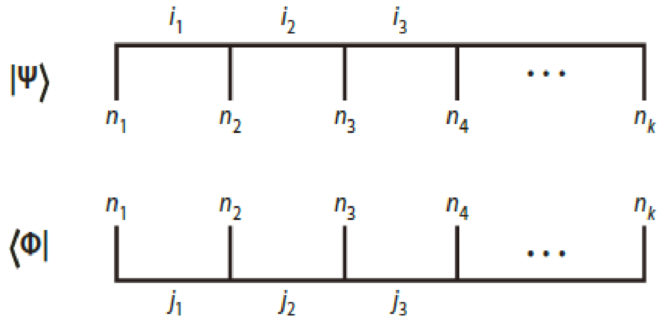
General form

$$C_{i_1 i_2 \dots i_N} = C^{i_1}_{\alpha_1} C^{i_2}_{\alpha_1 \alpha_2} \dots C^{i_N}_{\alpha_N}$$

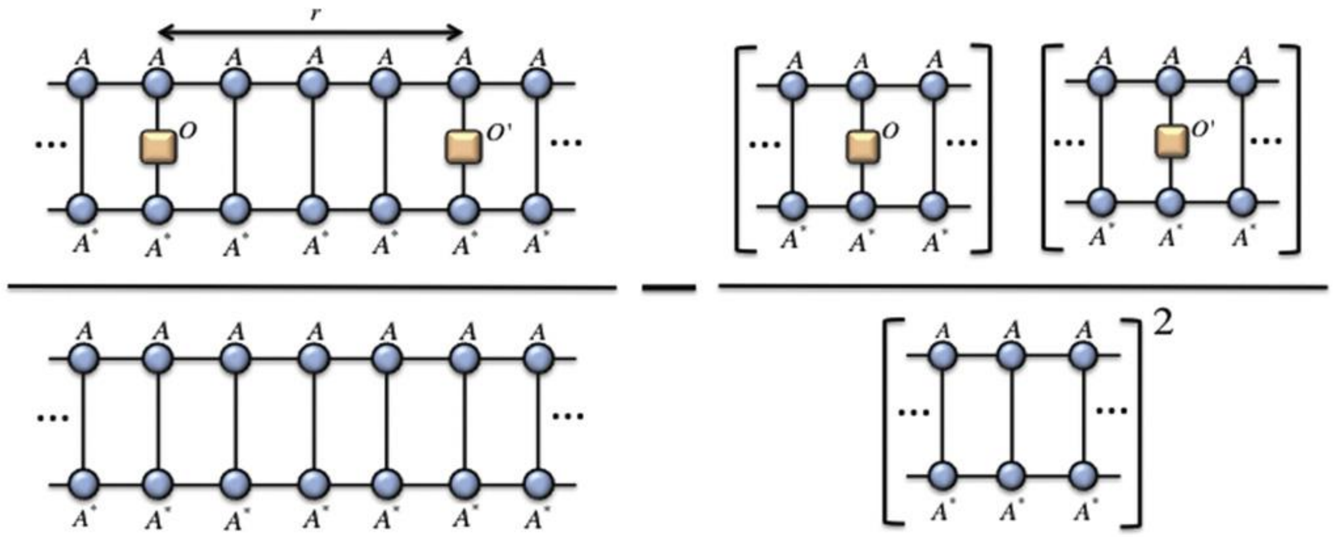
$$C^{n_1 n_2 n_3 n_4}_{DMRG} = \sum_{i_1 i_2 i_3 \dots}^{M} A^{n1}_{i_1} A^{n2}_{i_1 i_2} A^{n3}_{i_2 i_3} A^{n4}_{i_3} i_4 \dots$$

Matrix product states

Contraction



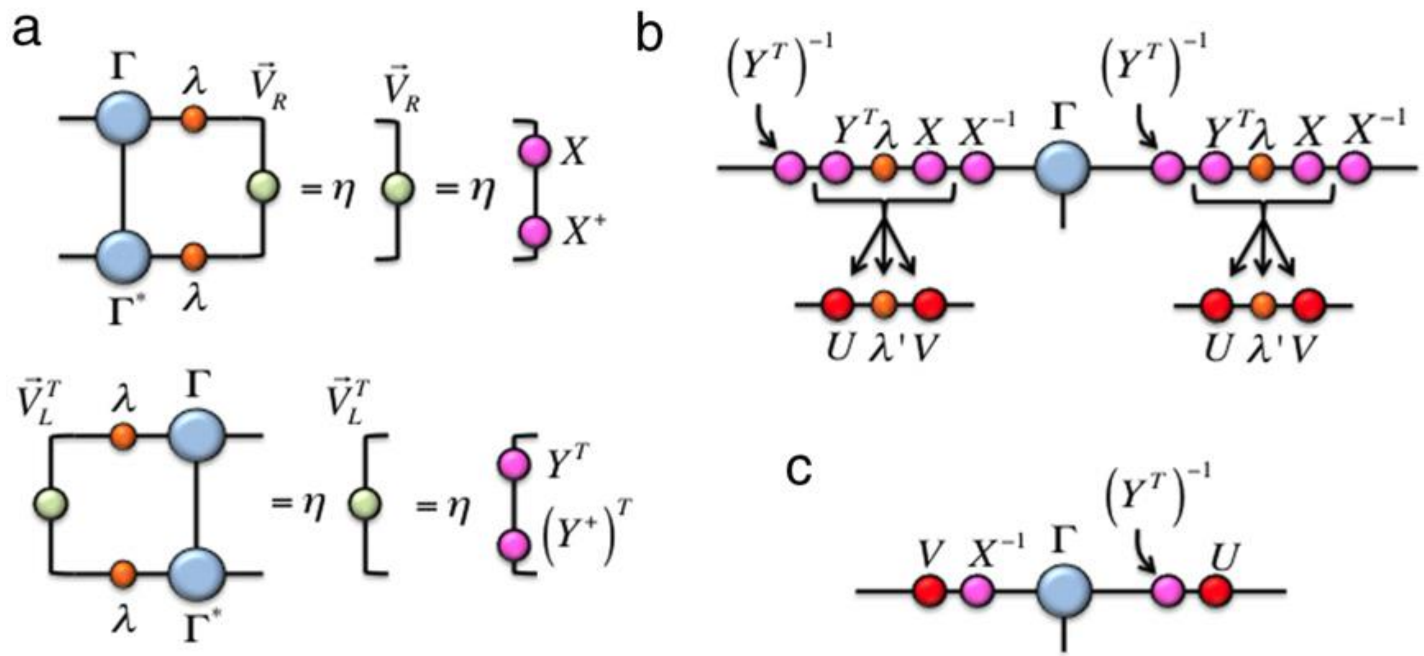
Correlation



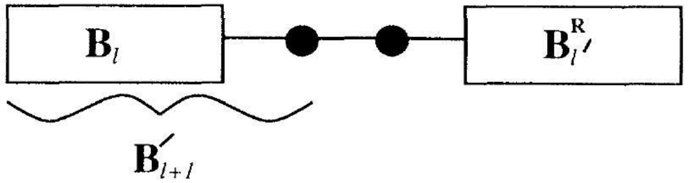
Matrix product states

1D Infinite chain and SVD

Same method for DMRG



DMRG



$$|B_l\rangle \otimes |i\rangle \qquad \langle i| \otimes \langle B^R|$$

SVD for density matrix

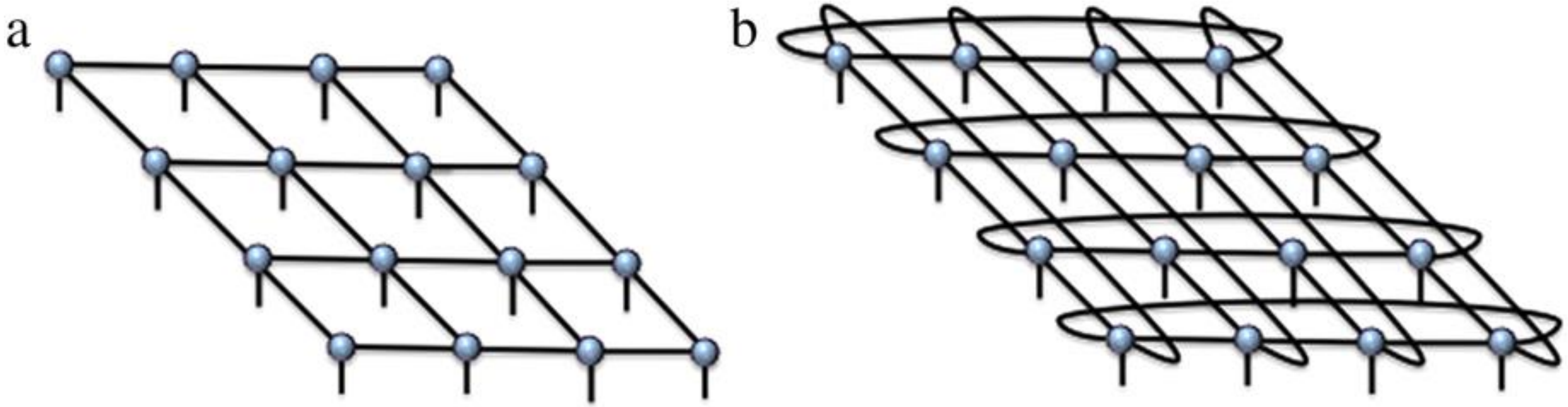
$$|B_{l+1}\rangle$$

Renormalization: reduce to certain rank.

Absorb only onsite for accuracy

2D tensor network

Projected entangled pair states (PEPS)



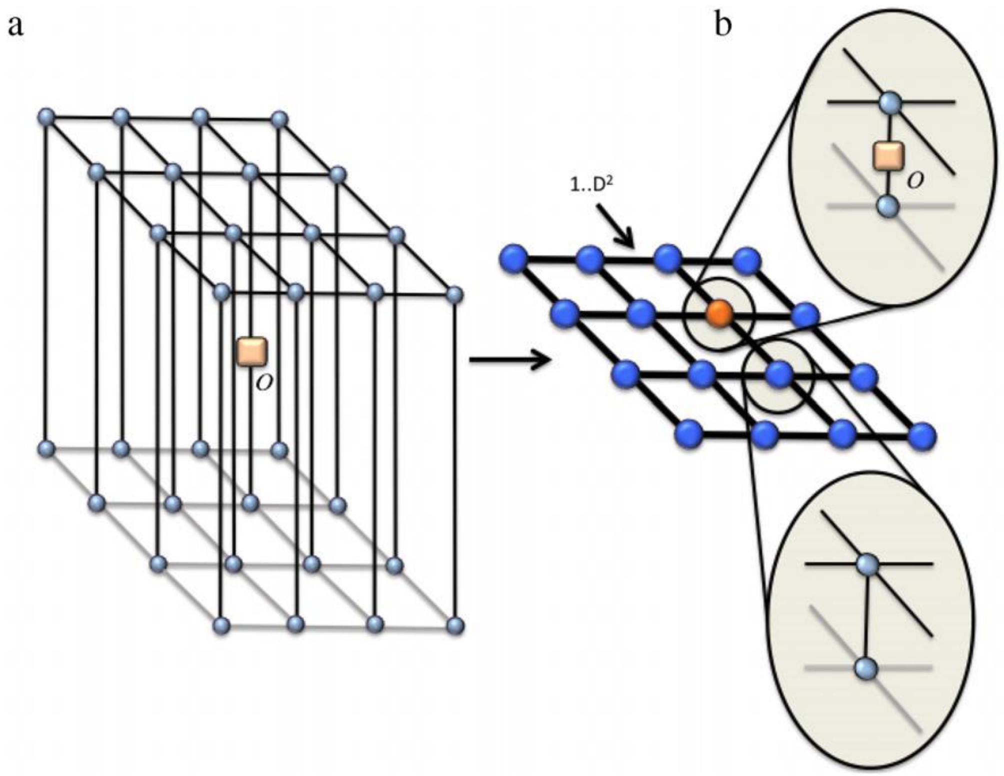
No canonical form

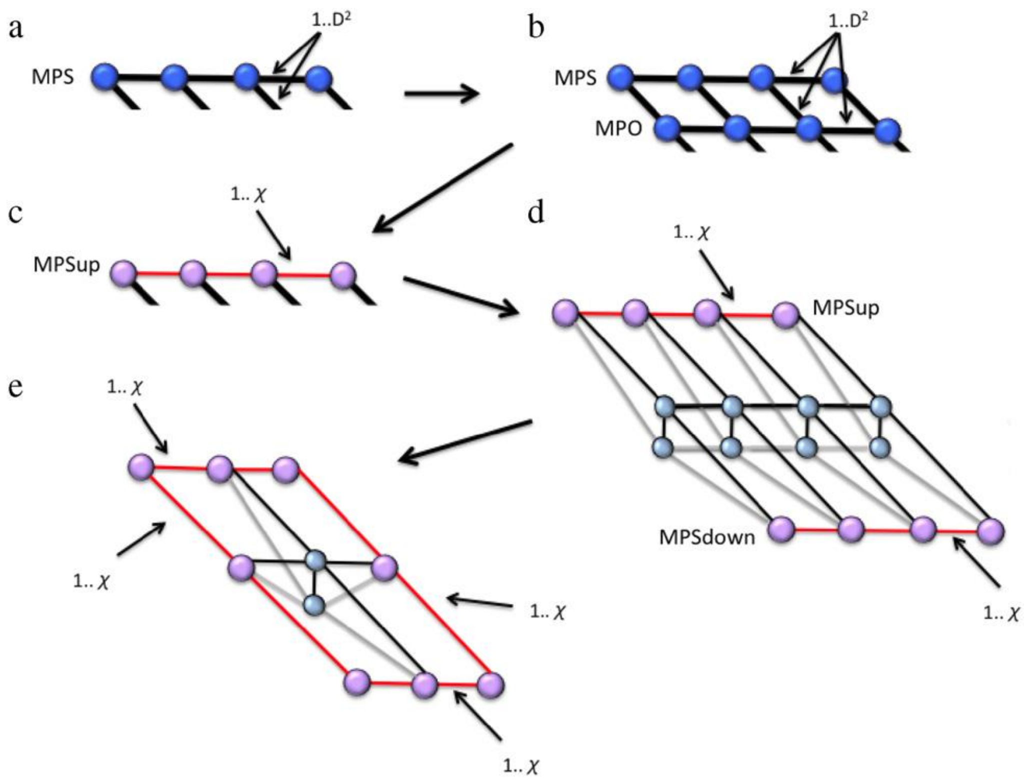
Long-term correlation

O(e^N) complexity to calculate contraction: NP-hard

PEPs and simple examples

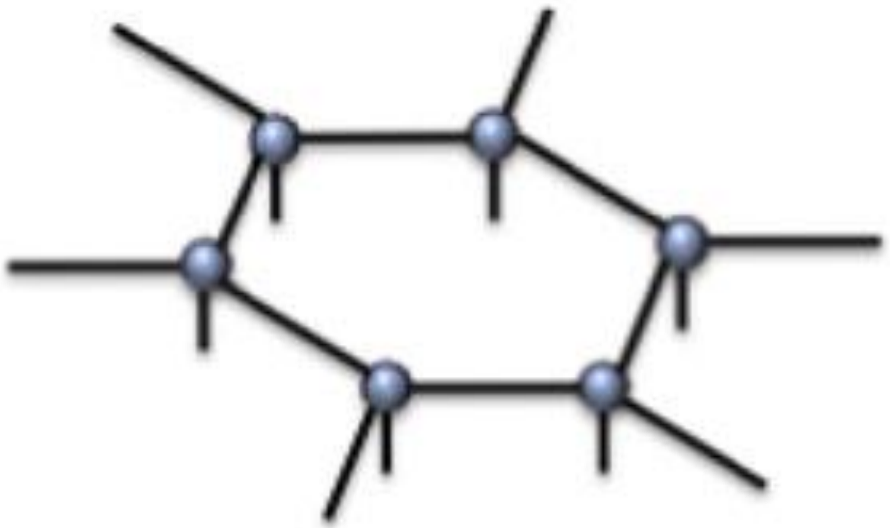
Boundary MPS methods

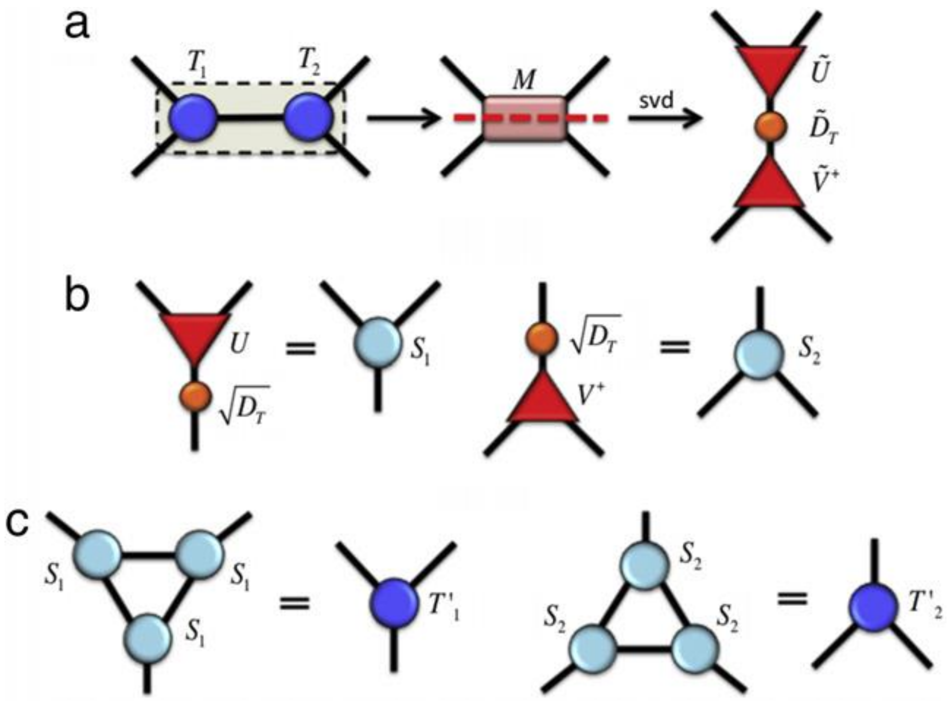


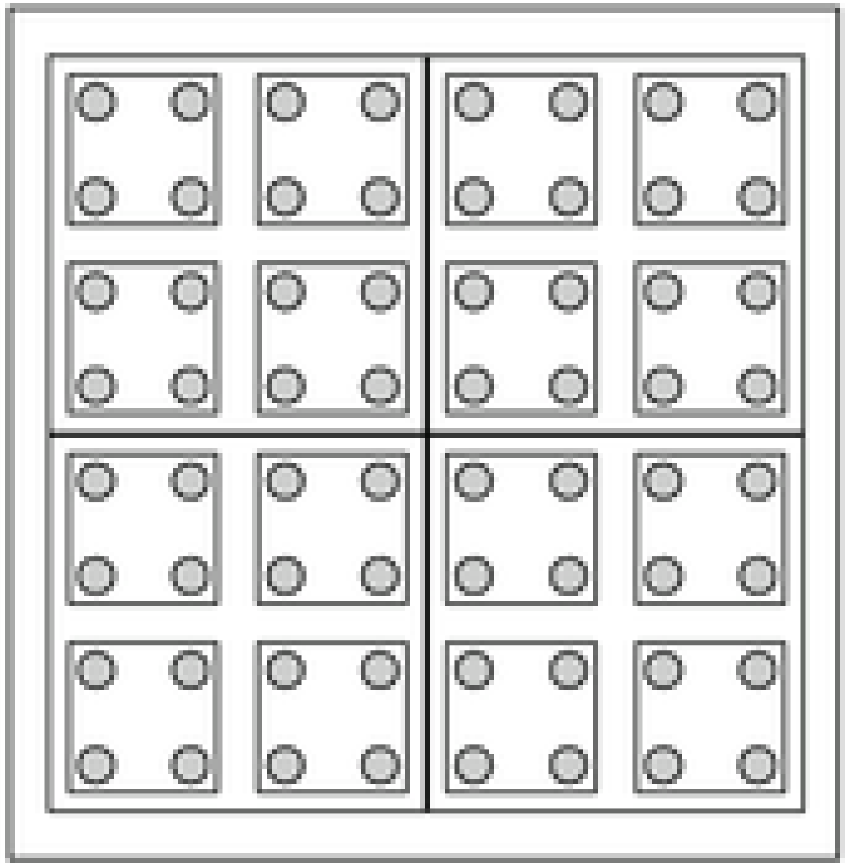


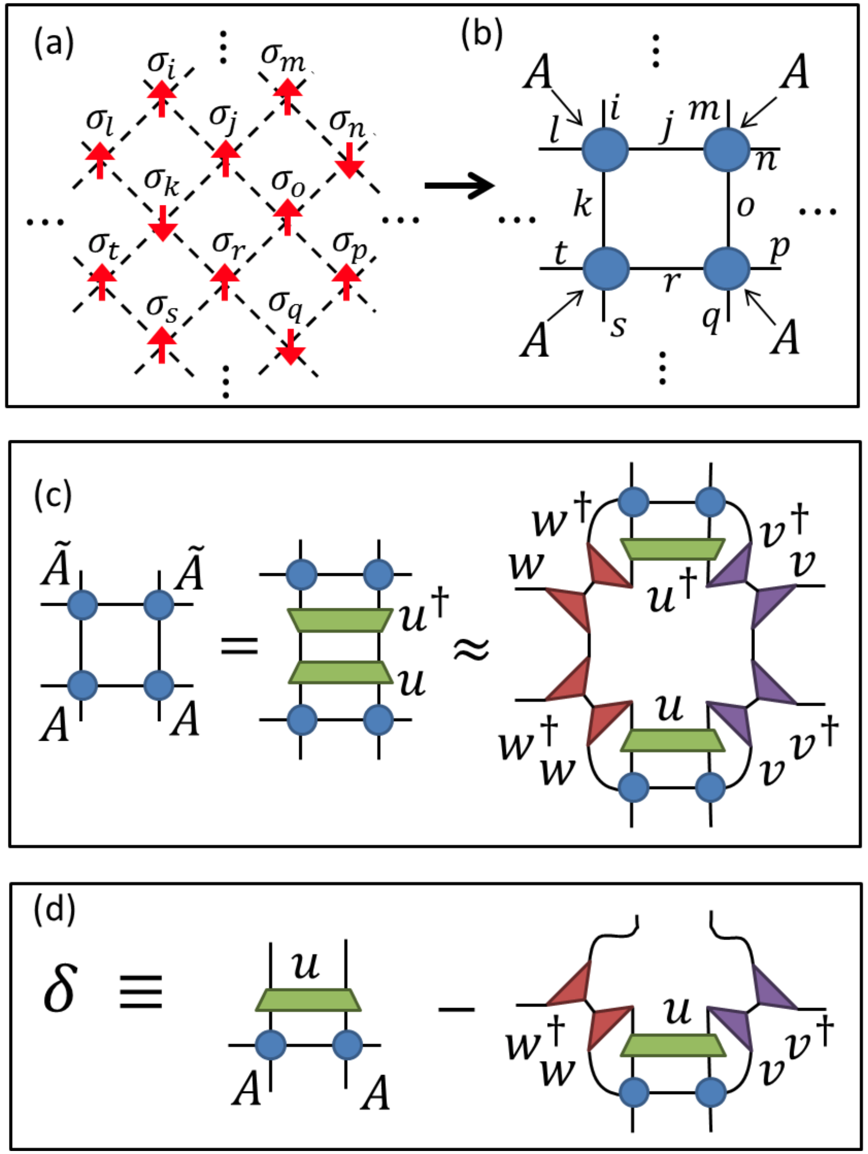
PEPs and simple examples

Tensor renormalization group









(a)
(b)
(c)
(d)
$\delta \equiv$