Assignment 2
Frequently Asked Questions

Part I: Error Bar of Sample-Mean

How accurate is the random number, $\pi^{(Ntry)} = \overline{f}^{(Ntry)} = \frac{1}{Ntry}\sum_{i=1}^{Ntry} f(r_i)$, where $f(x) = 4/(1+x^2)$ & $r_i \in [0,1]$ is a uniform random number, *i.e.*, output of mean.c?

Brute force: Run the program many times (say *Nseed* = 100) and observe how spread the outcomes are: $\sigma_{\mathrm{BF}}^{(Ntry)} = \sqrt{\frac{1}{Nseed}\sum_{outer=1}^{Nseed}(\pi_i^{(Ntry)})^2 - \left(\frac{1}{Nseed}\sum_{outer=1}^{Nseed}\pi_i^{(Ntry)}\right)^2}$, *i.e.*, run stdv.c.

Disadvantage: How do we know how large *Nseed* should be? Also, it's slow for large sample size, *Ntry*.

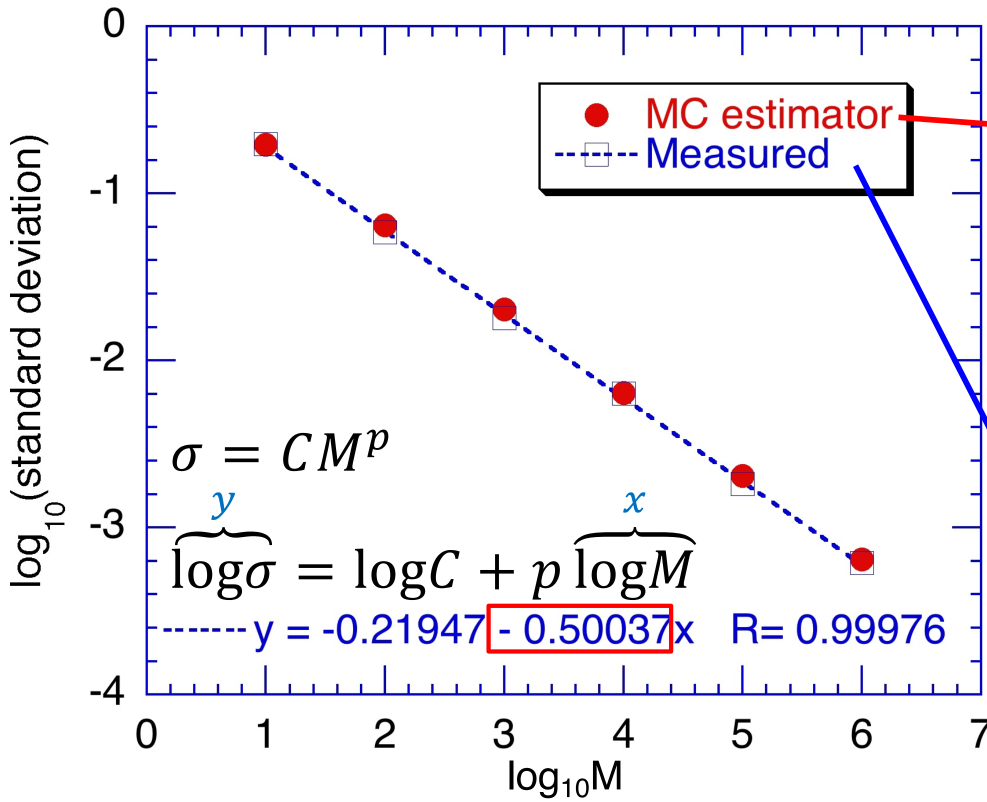
```
% time ./stdv    // Unix time command
Input the number of MC trials
100000000
Nry & stdv: 100000000 5.986821e-05
./stdv  71.68s user 0.32s system 95% cpu 1:15.19 total
```

Faster: Use the unbiased estimator of standard deviation, $\sigma_{\mathrm{estimator}}^{(Ntry)} = \sqrt{\frac{\overline{f^2}^{(Ntry)} - \left(\overline{f}^{(Ntry)}\right)^2}{Ntry-1}}$.

```
% time ./mean-stdv
Input the number of MC trials
100000000
MC estimate for PI & stdv: 100000000 3.141666 6.430597e-05
./mean-stdv  0.72s user 0.01s system 18% cpu 3.889 total
```

100x faster!

Is The Formula Correct?



$$\sigma_{\text{estimator}}^{(Ntry)} = \sqrt{\frac{\overline{f^2}^{(Ntry)} - \left(\overline{f}^{(Ntry)}\right)^2}{Ntry-1}}$$

Outputs from mean-stdv.c

$$\sigma_{\text{BF}}^{(Ntry)} = \sqrt{\frac{1}{Nseed}\sum_{outer=1}^{Nseed}(\pi_i^{(Ntry)})^2 - \left(\frac{1}{Nseed}\sum_{outer=1}^{Nseed}\pi_i^{(Ntry)}\right)^2}$$

Outputs from stdv.c

It is! Use the formula