Q. Proof of $|\partial(\xi,\zeta)/\partial(x,y)| = |\partial(x,y)/\partial(\xi,\zeta)|^{-1}$?

$$\left|\frac{\partial(\xi,\zeta)}{\partial(x,y)}\right|\left|\frac{\partial(x,y)}{\partial(\xi,\zeta)}\right|$$

$$= \left(\frac{\partial\xi}{\partial x}\frac{\partial\zeta}{\partial y} - \frac{\partial\xi}{\partial y}\frac{\partial\zeta}{\partial x}\right)\left(\frac{\partial x}{\partial\xi}\frac{\partial y}{\partial\zeta} - \frac{\partial x}{\partial\zeta}\frac{\partial y}{\partial\xi}\right)$$

$$= \frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\xi}\frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\zeta} - \frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\zeta}\frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\xi} - \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\zeta}\frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\xi} + \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\xi}\frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\zeta}$$

$$- \frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\xi}\frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\zeta} + \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\xi}\frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\zeta}\left(+\frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\xi}\frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\zeta} + \frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\xi}\frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\zeta}\right)$$

$$- \frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\zeta}\frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\xi} - \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\zeta}\frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\xi}\left(-\frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\xi}\frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\zeta} - \frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\xi}\frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\zeta}\right)$$

$$= \underbrace{\left(\frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\xi} + \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\xi}\right)}_{1}\underbrace{\left(\frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\zeta} + \frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\zeta}\right)}_{1} - \underbrace{\left(\frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\zeta} + \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\zeta}\right)}_{0}\underbrace{\left(\frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\xi} + \frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\xi}\right)}_{0}$$

$$= 1$$

Here, we have used.

$$\frac{\partial\xi}{\partial\xi} = \frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\xi} + \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\xi} = 1$$

$$\frac{\partial\zeta}{\partial\zeta} = \frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\zeta} + \frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\zeta} = 1$$

$$\frac{\partial\xi}{\partial\zeta} = \frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\zeta} + \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\zeta} = 0$$

$$\frac{\partial\zeta}{\partial\xi} = \frac{\partial\zeta}{\partial x}\frac{\partial x}{\partial\xi} + \frac{\partial\zeta}{\partial y}\frac{\partial y}{\partial\xi} = 0$$

☺ The change of $\xi(x,y)$ corresponding to $(x,y) \to (x+dx, y+dy)$ is

$$d\xi = \frac{\partial\xi}{\partial x}dx + \frac{\partial\xi}{\partial y}dy$$

~~If the changes, dx & dy, arise from $\xi \to \xi + d\xi$~~ while keeping ζ constant, then

$$dx = \frac{\partial x}{\partial\xi}d\xi \;;\quad dy = \frac{\partial y}{\partial\xi}d\xi$$

$$\therefore d\xi = \left(\frac{\partial\xi}{\partial x}\frac{\partial x}{\partial\xi} + \frac{\partial\xi}{\partial y}\frac{\partial y}{\partial\xi}\right)d\xi$$