# Minimal Linear Algebra

- **Matrix**:

$$\underset{m\times n}{\mathbf{A}} = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ m \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix} \end{array}$$

- **Matrix multiplication:**

$$(\underset{m\times l}{\overset{m\times n\ \ n\times l}{AB}})_{ij} \equiv \sum_{k=1}^{n} A_{ik}B_{kj} = \underset{\text{inner product}}{\overset{i\text{-th row vector} \quad j\text{-th column vector}}{A_{i*} \bullet B_{*j}}}$$

- **Inverse matrix:** Let $A$ be an $n \times n$ matrix, then

$$AA^{-1} = I \quad I_{ij} = \delta_{ij} = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j) \end{cases}$$

where $I$ is the identity matrix.

- **Determinant:** Let A be an $n \times n$ matrix and, for permutation $P$, sign($P$) = +1 (even permutation) and −1 (odd permutation), then

$$\det A = \sum_{P} \text{sign}(P) A_{1P(1)} A_{2P(2)} \cdots A_{nP(n)}$$

(Example: $n = 2$)

$$\det A = \underset{(12)}{A_{11}A_{22}} - \underset{(21)}{A_{12}A_{21}}$$

- **Inverse matrix:**

$$\left(A^{-1}\right)_{ij} = \frac{C_{ji}}{\det A}$$

where $C_{ji}$ is the co-factor (the determinant of an ($n$–1) × ($n$–1) matrix formed by striking out the $j$-th row and the $i$-th column)

$$C_{ji} = (-1)^{j+i} \det \begin{bmatrix} & & A_{1i} & & \\ & & \vdots & & \\ A_{j1} & \cdots & A_{ji} & \cdots & A_{jn} \\ & & \vdots & & \\ & & A_{ni} & & \end{bmatrix}$$

(Example: $n = 2$)

$$A^{-1} = \frac{1}{A_{11}A_{22} - A_{12}A_{21}} \begin{bmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{bmatrix}$$