Assignment 1 Sketch

If you solve the **characteristic equation** [Eq. (4)] right, you get these two eigenvalue-eigenvector pairs.

1st eigenpair

$$\begin{cases} \begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \overbrace{(a+b)}^{\epsilon_+} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{matrix} u_+ \\ v_+ \end{matrix} \\ \begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \overbrace{(a-b)}^{\epsilon_-} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{matrix} u_- \\ v_- \end{matrix} \end{cases} \Leftrightarrow \overbrace{\begin{pmatrix} a & b \\ b & a \end{pmatrix}}^{A} \overbrace{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}^{U} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \overbrace{\begin{pmatrix} a+b & 0 \\ 0 & a-b \end{pmatrix}}^{D}$$

2nd eigenpair

Matrix form: $AU = UD$

Or

$$AU = U \begin{pmatrix} a+b & 0 \\ 0 & a-b \end{pmatrix}$$

$\times U^{-1}$

$$A \overbrace{UU^{-1}}^{I} = U \begin{pmatrix} a+b & 0 \\ 0 & a-b \end{pmatrix} U^{-1}$$

Eigen decomposition:
$A = UDU^{-1}$

Now n-th power by **telescoping** [Eq. (9)]

$$A^n = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} (a+b)^n & 0 \\ 0 & (a-b)^n \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1}$$

Don't stop here! You have to finish inverting and multiplying matrices to get the final closed-form expression.

$$A^n = UD \overbrace{U^{-1}\,U}^{I} DU^{-1} \cdots UD \overbrace{U^{-1}\,U}^{I} DU^{-1} = UD^nU^{-1}$$

Simple Math Take Home

- **Exponential function: Solution of first-order differential equation**

$$\frac{df}{dt} = \Gamma f \quad \Longrightarrow \quad f(t) = \exp(\Gamma t) f(0)$$

- **What if Γ is a matrix? Power (or any function through Taylor expansion) of a square matrix can be computed through eigen decomposition**

- **Explain your math in your own words (what that means? *cf*. Feynman's bird)**

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