Viterbi Algorithm and Beyond

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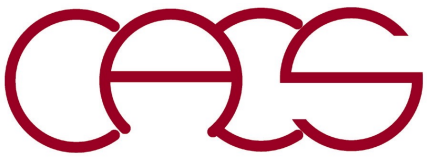


Top 10 Algorithms in History

In putting together this issue of *Computing in Science & Engineering*, we knew three things: it would be difficult to list just 10 algorithms; it would be fun to assemble the authors and read their papers; and, whatever we came up with in the end, it would be controversial. We tried to assemble the 10 algorithms with the greatest influence on the development and practice of science and engineering in the 20th century. Following is our list (here, the list is in chronological order; however, the articles appear in no particular order):

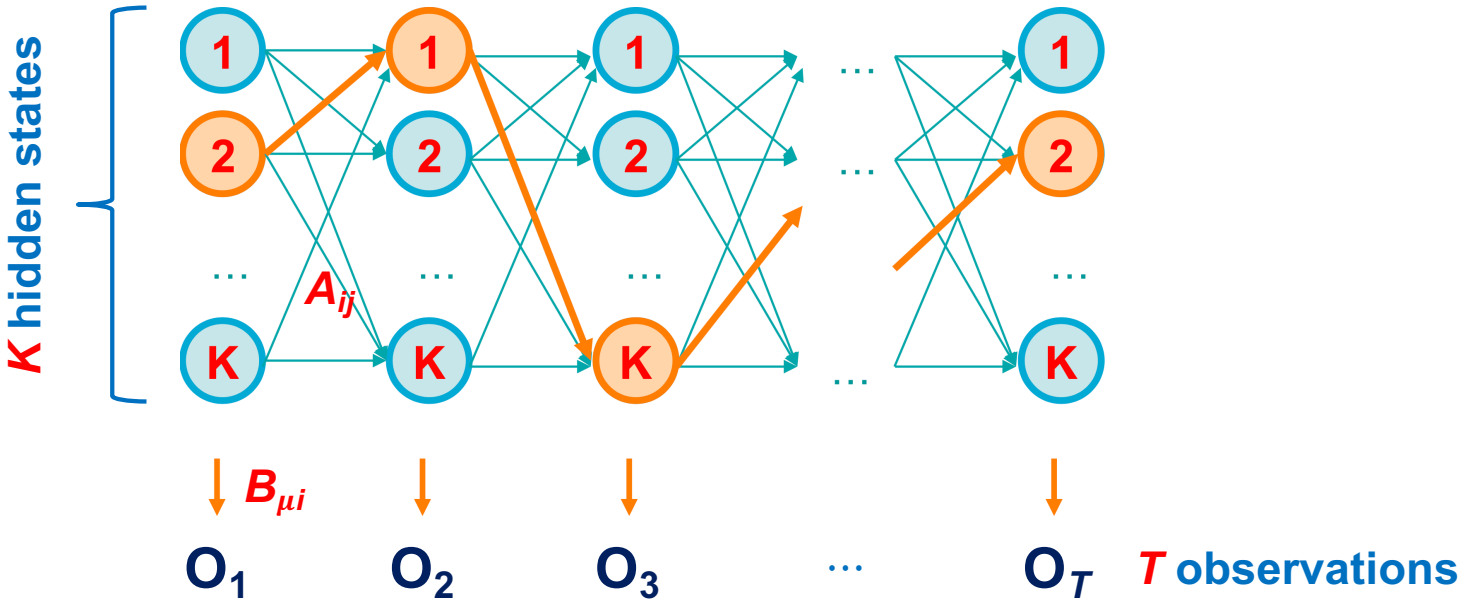
- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

PHYS 516
CSCI 596
CSCI 653

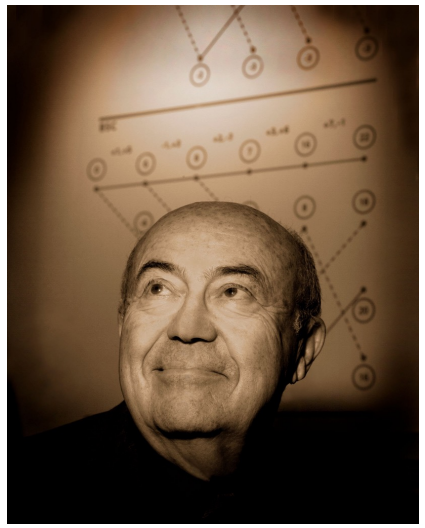
IEEE CiSE, Jan/Feb (2000)

Top 1 Algorithm in USC History?

- **Viterbi algorithm: Given a hidden Markov model (a set of K hidden states that transition between each other with transition-probability matrix A_{ij} & emit one of the observables with emission probability $B_{\mu i}$) & a sequence of T observations, determine the most likely state-transition path**



USC Viterbi School of Engineering



Andrew Viterbi

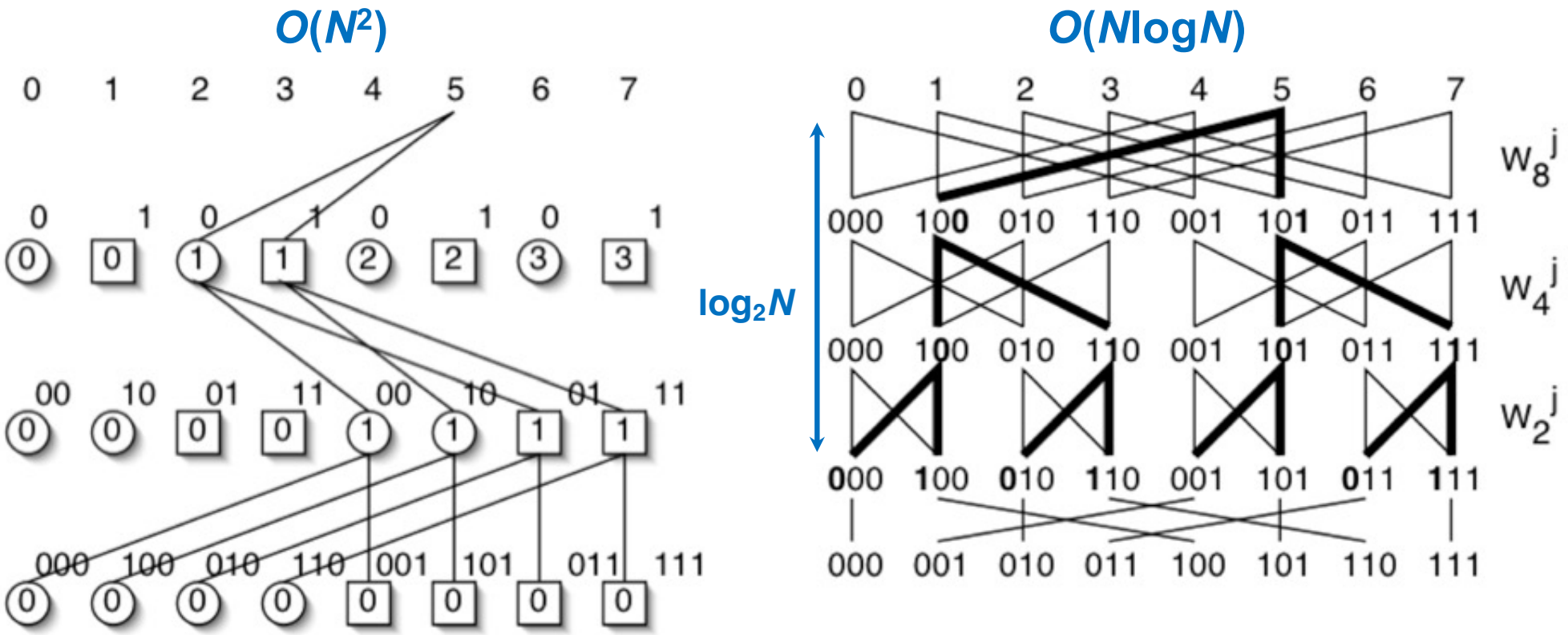
- **$O(K^T)$ way: Enumerate probability for all K^T paths**
- **$O(K^2T)$ Viterbi algorithm: Dynamic programming that recursively builds up necessary information for increasing observation time $t = 1,...,T$**

https://en.wikipedia.org/wiki/Viterbi_algorithm

***cf.* Waterman-Smith algorithm, RSAdleman cryptography**

cf. Fast Fourier Transform (FFT)

- **FFT algorithm reuses many overlapping path segments among *N* divide-&conquer trees to reduce the $O(N^2)$ brute-force evaluations to $O(N\log N)$**



Analogy is mother of invention!

Have Fun Beyond Assignments

- **"If you decide you don't have to get A's, you can learn an enormous amount in college."** ***Isidor Isaac Rabi***

- **In this course, we also learned:**
  1. **Singular value decomposition (SVD) & Cholesky decomposition for dimensionality reduction (low-rank approximation)**
  2. **Krylov subspace method: Lanczos algorithm for $O(N)$ eigensolver**

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And get A!

- **Extension: randomized low-rank matrix decomposition?**

Murray *et al.*, *arXiv*:2302.11474 ('23)